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IDEAL GASES

1. The pressure p of an ideal gas is given by the expression

$$p = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle .$$

(a) Explain the meaning of the symbol $\langle c^2 \rangle$.

....

.... [2]

(b) The ideal gas has a density of 2.4 kgm^{-3} at a pressure of 2.0×10^5 Pa and a temperature of 300 K.

(i) Determine the root-mean-square (r.m.s.) speed of the gas atoms at 300 K.

r.m.s. speed = ... m s^{-1} [3]

(ii) Calculate the temperature of the gas for the atoms to have an r.m.s. speed that is twice that calculated in **(i)**.

temperature = ... K [3]

2. (a) (i) The kinetic theory of gases leads to the equation

$$\tfrac{1}{2} m\langle c^2 \rangle = \tfrac{3}{2} kT.$$

Explain the significance of the quantity $\tfrac{1}{2} m\langle c^2 \rangle$.

…………………...

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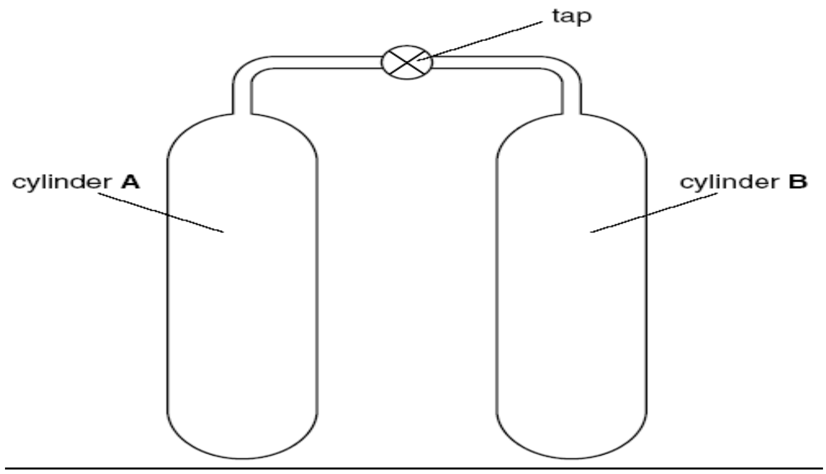
(ii) Use the equation to suggest what is meant by the absolute zero of temperature.

....

....[3]

(b) Two insulated gas cylinders **A** and **B** are connected by a tube of negligible volume, as shown in Figure.



Each cylinder has an internal volume of $2.0 \times 10^{-2} m^3$. Initially, the tap is closed and cylinder **A** contains 1.2 mol of an ideal gas at a temperature of 37 °C. Cylinder **B** contains the same ideal gas at pressure 1.2×10^5 Pa and temperature 37 °C.

(i) Calculate the amount, in mol, of the gas in cylinder **B**.

amount = ... mol

(ii) The tap is opened and some gas flows from cylinder **A** to cylinder **B**. Using the fact that the total amount of gas is constant, determine the final pressure of the gas in the cylinders.

pressure = ... Pa

[6]

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3. If an object is projected vertically upwards from the surface of a planet at a fast enough speed, it can escape the planet's gravitational field. This means that the object can arrive at infinity where it has zero kinetic energy. The speed that is just enough for this to happen is known as the escape speed.

(a) (i) By equating the kinetic energy of the object at the planet's surface to its total gain of potential energy in going to infinity, show that the escape speed *v* is given by

$$v^2 = \frac{2GM}{R},$$

where *R* is the radius of the planet and *M* is its mass.

(ii) Hence show that

$$v^2 = 2Rg,$$

where *g* is the acceleration of free fall at the planet's surface.

[3]

(b) The mean kinetic energy *E*k of an atom of an ideal gas is given by

$$E_k = \frac{3}{2}kT,$$

where *k* is the Boltzmann constant and *T* is the thermodynamic temperature.
Using the equation in **(a)(ii)**, estimate the temperature at the Earth's surface such that helium atoms of mass 6.6×10^{-27} kg could escape to infinity.
You may assume that helium gas behaves as an ideal gas and that the radius of Earth is 6.4×10^{6}m.

temperature = ... K [4]

4. (a) Outline an experiment which demonstrates that the molecules in a gas are in perpetual random motion.

....

....

....

....

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……………………………………………………………………………………………………

…………………………………………………………………………………………………… [3]

(b) The pressure *p* of an ideal gas is given by both of the following equations.

$$p = \frac{Nm < c^2 >}{3V} \qquad\qquad p = \frac{NkT}{V}$$

(i) Use the equations to show that the average translational kinetic energy of a molecule is proportional to the temperature *T*.

[3]

(ii) Calculate the average kinetic energy of a molecule of an ideal gas at a temperature of 27 °C.

kinetic energy = ... J [2]

(iii) Explain why the answer to **(ii)** is independent of the mass of the gas molecules.

....

....

.... [2]

(iv) A laboratory contains 2600 mol of air at a temperature of 27 °C. Calculate the total kinetic energy of all the molecules of air in the laboratory.

kinetic energy = ... J [2]

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IDEAL GASES

5.

(a) The equation

$$pV = constant \times T$$

relates the pressure p and volume V of a gas to its kelvin (thermodynamic) temperature T.

State two conditions for the equation to be valid.

1. ....

....

2. ....

.... [2]

(b) A gas cylinder contains $4.00 \times 10^4\,\text{cm}^3$ of hydrogen at a pressure of $2.50 \times 10^7\,\text{Pa}$ and a temperature of 290 K.

The cylinder is to be used to fill balloons. Each balloon, when filled, contains $7.24 \times 10^3\,\text{cm}^3$ of hydrogen at a pressure of $1.85 \times 10^5\,\text{Pa}$ and a temperature of 290 K.

Calculate, assuming that the hydrogen obeys the equation in **(a)**,

(i) the total amount of hydrogen in the cylinder,

amount = ………………………….. mol [3]

(ii) the number of balloons that can be filled from the cylinder.

number = ………………………….. [3]

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IDEAL GASES

6.

(a) Explain what is meant by the Avogadro constant.

....

....

.... [2]

(b) Argon-40 ($^{40}_{18}Ar$) may be assumed to be an ideal gas.
A mass of 3.2 g of argon-40 has a volume of 210 cm^3 at a temperature of 37 °C.

Determine, for this mass of argon-40 gas,

(i) the amount, in mol,

amount = ... mol [1]

(ii) the pressure,

pressure = ... Pa [2]

(iii) the root-mean-square (r.m.s.) speed of an argon atom.

r.m.s. speed = ... $m s^{-1}$ [3]

{*Q. 2/41 & 42 Variant/ June 2014*}

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IDEAL GASES

7.

(a) State what is meant by an *ideal gas*.

....

....

....

....[3]

(b) Two cylinders A and B are connected by a tube of negligible volume, as shown in Fig. 2.1.

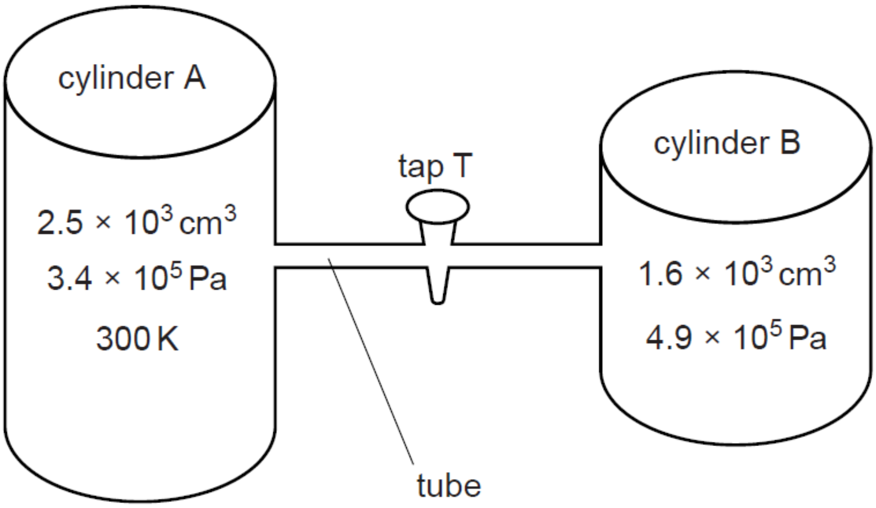

Fig. 2.1

Initially, tap T is closed. The cylinders contain an ideal gas at different pressures.

(i) Cylinder A has a constant volume of $2.5 \times 10^3\,cm^3$ and contains gas at pressure $3.4 \times 10^5\,Pa$ and temperature 300 K.

Show that cylinder A contains 0.34 mol of gas.

[1]

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(ii) Cylinder B has a constant volume of $1.6 \times 10^3\,cm^3$ and contains 0.20 mol of gas. When tap T is opened, the pressure of the gas in both cylinders is 3.9×10^5 Pa. No thermal energy enters or leaves the gas.

Determine the final temperature of the gas.

temperature = ... K [2]

(c) From Thermal properties of Materials chapter [3]

{*Q. 2/41 & 43 Variant/ June 2013*}

8.

(a) An ideal gas is assumed to consist of atoms or molecules that behave as hard, identical spheres that are in continuous motion and undergo elastic collisions.

State two further assumptions of the kinetic theory of gases.

1. ....

....

2. ....

....

[2]

(b) Helium-4 ($^4_2$He) may be assumed to be an ideal gas.

(i) Show that the mass of one atom of helium-4 is 6.6×10^{-24} g.

[1]

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(ii) The mean kinetic energy E_K of an atom of an ideal gas is given by the expression

$$E_K = \frac{3}{2}kT.$$

Calculate the root-mean-square (r.m.s.) speed of a helium-4 atom at a temperature of 27 °C.

r.m.s. speed = ... $m\,s^{-1}$ [3]

{*Q. 2/41 & 43 Variant/ June 2016*}

9.

(a) State what is meant by

(i) the Avogadro constant N_A,

....

.... [1]

(ii) the mole.

....

.... [2]

(b) A container has a volume of $1.8 \times 10^4\,cm^3$.

The ideal gas in the container has a pressure of $2.0 \times 10^7\,Pa$ at a temperature of 17 °C.

Show that the amount of gas in the cylinder is 150 mol.

[1]

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(c) Gas molecules leak from the container in **(b)** at a constant rate of $1.5 \times 10^{19}\,s^{-1}$.
The temperature remains at 17 °C.
In a time *t*, the amount of gas in the container is found to be reduced by 5.0%.

Calculate

(i) the pressure of the gas after the time *t*,

pressure = ... Pa [2]

(ii) the time *t*.

t = ... s [3]

{*Q. 2/42 Variant/ June 2016*}

Marking Key

Q.1 to Q. 5) To be practiced in class

Q. 6. {*Ref.: Q. 2/41 & 42 Variant/ June 2014*}

(a) the number of atoms — M1
in 12 g of carbon-12 — A1 [2]

(b) (i) amount = 3.2/40
= 0.080 mol — A1 [1]

(ii) $pV = nRT$
$p \times 210 \times 10^{-6} = 0.080 \times 8.31 \times 310$ — C1
$p = 9.8 \times 10^5$ Pa — A1 [2]
(do not credit if T in °C not K)

(iii) *either* $pV = 1/3 \times Nm <c^2>$
$N = 0.080 \times 6.02 \times 10^{23}$ $(= 4.82 \times 10^{22})$
<u>and</u> $m = 40 \times 1.66 \times 10^{-27}$ $(= 6.64 \times 10^{-26})$ — C1
$9.8 \times 10^5 \times 210 \times 10^{-6} = 1/3 \times 4.82 \times 10^{22} \times 6.64 \times 10^{-26} \times <c^2>$ — C1
$<c^2> = 1.93 \times 10^5$
$c_{RMS} = 440\ m\ s^{-1}$ — A1 [3]

or $Nm = 3.2 \times 10^{-3}$ — (C1)
$9.8 \times 10^5 \times 210 \times 10^{-6} = 1/3 \times 3.2 \times 10^{-3} \times <c^2>$ — (C1)
$<c^2> = 1.93 \times 10^5$
$c_{RMS} = 440\ m\ s^{-1}$ — (A1)

or $1/2\ m<c^2> = 3/2\ kT$ — (C1)
$1/2 \times 40 \times 1.66 \times 10^{-27} <c^2> = 3/2 \times 1.38 \times 10^{-23} \times 310$ — (C1)
$<c^2> = 1.93 \times 10^5$
$c_{RMS} = 440\ m\ s^{-1}$ — (A1)

*(if T in °C not K award max 1/3, unless already penalised in **(b)(ii)**)*

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IDEAL GASES

7. {Ref.: *Q. 2/41 & 43 Variant/ June 2013*}

(a) obeys the equation pV = constant × T or $pV = nRT$ — M1
p, V and T explained — A1
at all values of p, V and T/fixed mass/n is constant — A1 [3]

(b) (i) $3.4 \times 10^5 \times 2.5 \times 10^3 \times 10^{-6} = n \times 8.31 \times 300$ — M1
n = 0.34 mol — A0 [1]

(ii) for total mass/amount of gas
$3.9 \times 10^5 \times (2.5 + 1.6) \times 10^3 \times 10^{-6} = (0.34 + 0.20) \times 8.31 \times T$ — C1
T = 360 K — A1 [2]

8. {*Ref.: Q. 2/41 & 43 Variant/ June 2016*}

(a) e.g. time of collisions negligible compared to time between collisions

no intermolecular forces (except during collisions)

random motion (of molecules)

large numbers of molecules

(total) volume of molecules negligible compared to volume of containing vessel
or
average/mean separation large compared with size of molecules

any two — B2 [2]

(b) (i) mass = 4.0 / (6.02×10^{23}) = 6.6×10^{-24} g
or
mass = $4.0 \times 1.66 \times 10^{-27} \times 10^3 = 6.6 \times 10^{-24}$ g — B1 [1]

(ii) $\frac{3}{2}kT = \frac{1}{2}m\langle c^2\rangle$ — C1

$\frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = \frac{1}{2} \times 6.6 \times 10^{-27} \times \langle c^2\rangle$

$\langle c^2\rangle = 1.88 \times 10^6$ ($m^2 s^{-2}$) — C1

r.m.s. speed = 1.4×10^3 m s^{-1} — A1 [3]

9. {*Ref.: Q. 2/42 Variant/ June 2016*}

(a) (i) number of <u>atoms/nuclei</u> in 12 g of carbon-12 — B1 [1]

(ii) amount of substance — M1

containing N_A (or 6.02×10^{23}) particles/molecules/atoms
or
which contains the same number of particles/atoms/molecules as there are atoms in 12 g of carbon-12 — A1 [2]

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IDEAL GASES

(b) $pV = nRT$

$2.0 \times 10^7 \times 1.8 \times 10^4 \times 10^{-6} = n \times 8.31 \times 290$, so n = 149 mol or 150 mol — A1 [1]

(c) (i) V and T constant and so pressure reduced by 5.0%
pressure = $0.95 \times 2.0 \times 10^7$ — C1

or

calculation of new n (= 142.5 mol) and correct substitution into $pV = nRT$ — (C1)

pressure = 1.9×10^7 Pa — A1 [2]